What you'll Learn About

- Calculus using Parametric Equations

Convert the following parametric equations into a Cartesian Equation.
Then find the first derivative and 2nd derivative.

$y =$

A) $x = 4t$ $y = t^2$ at $t = 1$

$\frac{x}{4} = t$ $y = \left(\frac{x}{4}\right)^2$

$y = \frac{1}{16}x^2$

$\left.\frac{dy}{dx}\right|_{x=4} = \frac{1}{8}x = \frac{1}{8}(4) = \frac{1}{2}$

$\frac{d^2y}{dx^2} = \frac{1}{8}$

Find the first derivative and 2nd derivative of the parametric curve in terms of t.

B) $x = 4t$ $y = t^2$ at $t = 1$

$\frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{dy}{dx}$

$\frac{dx}{dt} = 4$ $\frac{dy}{dt} = 2t$

$\left.\frac{dy}{dx}\right|_{t=1} = \boxed{\frac{2t}{4}} = \frac{2}{4} = \frac{1}{2}$

$\frac{dy}{dx} = \frac{1}{2}t$

$\frac{d^2y}{dx^2} = \frac{1}{2}\left(\frac{dt}{dx}\right)$

$= \frac{1}{2}\left(\frac{1}{4}\right)$

$= \frac{1}{8}$

Find the first derivative and 2nd derivative of the parametric curve in terms of t

$\frac{dt}{dx} = x$

16. $x = \ln(5t)$ $y = e^{5t}$

$\frac{dx}{dt} = \frac{1}{(5t)\ln e}\cdot(5) = \frac{1}{t}$ $\frac{dy}{dt} = e^{5t}\cdot \ln e \cdot 5 = 5e^{5t}$

$\frac{dy}{dx} = \frac{5e^{5t}}{\left(\frac{1}{t}\right)} = (5t)e^{5t}$

$\frac{d^2y}{dx^2} = (5t)\left(e^{5t}\cdot 5\frac{dt}{dx}\right) + \left(e^{5t}\right)\left(5\frac{dt}{dx}\right)$

$= (5t)(e^{5t}\cdot 5t) + e^{5t}\cdot 5t$

$25t^2e^{5t} + 5te^{5t}$

Determine the leftmost point on the parametric curve between [-2, 3]

18. $\boxed{x = t^2 + 2t}$ $\boxed{y = t^2 - 2t + 3}$ $t=-2$ to $t=3$

Abs Max/min

1) Critical Points
   - derivative = 0
   - derivative und
2) Plug endpts and C.P. back into original

$\frac{dx}{dt} = 2t + 2$

$0 = 2t + 2$

$-1 = t$

$x(-2) = (-2)^2 + 2(-2) = 0$

$\boxed{x(-1) = (-1)^2 + 2(-1) = -1}$ - Leftmost point

$x(3) = 15$

$(-1, 6)$

26. Find the points at which the tangent line to the curve is horizontal or vertical.

$\boxed{x = -2 + 3\cos t}$ $\boxed{y = 1 + 3\sin t}$ $[0, 2\pi]$

Horizontal Tangent

$\frac{dy}{dx} = 0 \rightarrow \frac{dy}{dt} = 0$

$3\cos t = 0$

$\cos t = 0$

$t = \pi/2, 3\pi/2$

Vertical Tangent

$\frac{dy}{dx} = \text{UND} \rightarrow \frac{dx}{dt} = 0$

$-3\sin t = 0$

$\sin t = 0$

$t = 0, \pi, 2\pi$

$(1, 1)$ $(-5, 1)$

$t = \frac{\pi}{2}$

$x = -2 + 3\cos \pi/2$

$y = 1 + 3\sin \pi/2$

$(-2, 4)$

$t = \frac{3\pi}{2}$ $(-2, -2)$

$x = -2 + 3\cos 3\pi/2$

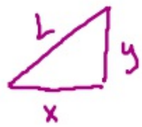
$y = 1 + 3\sin 3\pi/2$

28. Find the length of the curve.

$x = 3\sin t \quad y = \cos t \quad [0, \pi]$

$\frac{dx}{dt} = 3\cos t \quad \frac{dy}{dt} = -\sin t$

$L = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$



$L = \int_0^{\pi} \sqrt{(3\cos t)^2 + (-\sin t)^2}\, dt$

$= \int_0^{\pi} \sqrt{9\cos^2 t + \sin^2 t}$

Arc Length in Cartesian

$L = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$